Distributed Linear Block Coding for Cooperative Wireless Communications

Pierluigi Salvo Rossi, Athina P. Petropulu, Francesco Palmieri, and Giulio Iannello

***Abstract*—Performance of wireless communications systems is degraded by fading, typically combated via spatial diversity by multiple transmit and receiver antennas. Practical limitations may limit their use; thus, cooperative transmissions have been proposed to introduce diversity without the need for multiple antennas. We propose a novel cooperative scheme that performs distributed coding of user data. It needs each user transmit on one single channel and allows large flexibility between rate and diversity. A trade-off is increased complexity as more users are required to be decoded simultaneously by the base station. We present construction of the distributed code for different choices of rates and diversity orders by means of simple linear block codes, resulting in reduced complexity for decoding operations. Achieved diversity order is bounded by the minimum distance of the implemented distributed code. Performance of the scheme is studied via analytical bounds and numerical simulations.**

***Index Terms*—Coded cooperation, diversity, wireless communications.**

NOTATION*:* Bold upper (resp. lower) case letters denote matrices (resp. column vectors); $[.]^{\mathrm{T}}$ denotes transpose; "$+$" and "$\cdot$" denote sum and row-column product (both for real and modulo arithmetic); and $\mathrm{Diag}(\mathbf{x})$ denotes a diagonal matrix whose main diagonal is $\mathbf{x}$.

I. Introduction

FADING severely affects wireless-communications performance, causing large variations in signal strength as function of the user position. Diversity is a powerful technique against fading, used into spatial, temporal, and frequency domains [1], [2]. Spatial diversity is typically created via space–time coding with multiple antennas [3], but often user constraints confine the deployment of multiple antennas to the base station (BS). Cooperation has recently emerged to obtain spatial diversity while using a single antenna at user location [4]. Users share their single antennas (exploiting the free wireless connection) and transmit information on behalf of other users as well as their own information. This is more general than the relay scenario [5], for each user act simultaneously as source and relay. Various schemes have been proposed, such as *Amplify* (resp. *Decode*) *and Forward* in which users listen, amplify (resp. decode), and transmit signals from their partners [6], [7]. *Coded Cooperation* [8] is more efficient as channel coding replaces repetition: users propagate each one's codewords through different channels.

This letter proposes a new scheme that uses block coding and cooperation diversity, in which each user transmits on a different orthogonal channel and listens on the remaining ones. The information transmitted over each channel (code symbols) is a combination of locally-generated information (source symbols) and incoming estimated information from the partners (side symbols). We use the modulo sum for "fair" symbol combination, as it does not change the symbol alphabet and/or energy, allowing to focus on the advantage of cooperation. The proposed strategy may be viewed as an equivalent linear block code built out of joint source information from all the users. Classical coded cooperation requires each user to perform channel decoding and re-encoding, while our scheme does not require channel decoding at user location, but only demodulation and symbol detection.[1] This approach easily allows large flexibility to select data rates as users do not need to recover each other's coded information. The final task of channel decoding to recover source information from the code symbols is left to the BS. Our scheme allows to select the point of operation of the system exploiting the trade-off between diversity and rate, by increasing the number of cooperative users. The price is complexity as more users are required to be simultaneously decoded by the BS. We propose a code design for the cooperation group (CG) that results in reduced complexity for decoding operations at the BS.

Summarizing, we can say that: 1) Laneman *et al.* proposed cooperation with repetition and space-time coding; 2) Nosratinia *et al.* merged cooperation and channel coding; and 3) our scheme still implements cooperation and channel coding but does not require users to perform channel decoding (relegated to the BS only), resulting in simpler operations for the users.

II. Cooperative Scheme

A CG is composed by J cooperative users, each transmitting on a different channel. Sent symbols are received by each user and by the BS, chosen from an M-ary alphabet, combined in modulo-M arithmetic. The system is synchronous on symbol time (ST) and organized in frames of n ST. Each frame, each user transmits k source symbols via n code symbols (same rate $R = k/n$) carrying information also on side symbols.

For the ith user, denote source, side (from the jth user), and code symbols related to the ℓth ST, with $b_i(\ell)$, $s_j^{(i)}(\ell)$, and $c_i(\ell)$. At the ℓth ST of the generic frame, $\{b_i(1), \dots, b_i(k)\}$ and $\left\{s_j^{(i)}(1), \dots, s_j^{(i)}(\ell-1)\right\}_{j=1, j\neq i}^{J}$ are available at the ith user for $c_i(\ell)$ generation, i.e., all own symbols and those from the partners up to that time. In the following, we show how to achieve diversity for each user and contained complexity at

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[1]"Channel coding and decoding" means mapping discrete-symbols strings into codewords and vice-versa to protect transmission from errors [1], "symbol detection" the estimation of a single symbol in a digital transmission scheme.

the BS. The former is obtained via distribution of the source symbols on the available channels; the latter decomposes the Jn equations, describing the overall coding of the Jk source symbols to the BS, into J simpler n-equation groups, related to k source symbols. Simple cyclic operations (e.g., adding the side symbol from previous user) distribute source symbols over channels. The simple group will be described as

$$\mathbf{v}_i = \mathbf{A}\,\mathbf{u}_i\,, \quad 1 \le i \le J \tag{1}$$

with $\mathbf{A}$, $\mathbf{u}_i$, and $\mathbf{v}_i$ representing a linear block code (k, n), source and code symbols.[2] It is worth noticing that (1) presents same encoding (and same complexity for decoding at the BS) of a simple linear code on the single user, but due to cooperation, better performances are achieved in fading environments.

A. Basic Example

In order to present the proposed strategy, we consider the case of $J = n = 3$ and $k = 2$ with $M = 2$ [9]. Assume that during each frame, the single user behaves according to: 1) transmission in the first ST of its own first source symbol; 2) transmission in the second ST of the XOR-sum between its own second source symbol and the first ST side symbol from the previous user; and 3) transmission in the third ST of the XOR-sum between the second ST side symbol from the previous user and the first ST side symbol from the twice-previous user. Letting $\mu(i,n) = \begin{cases} \mathrm{mod}\,(i,n) & \mathrm{mod}\,(i,n) \neq 0 \\ n & \mathrm{mod}\,(i,n) = 0 \end{cases}$, the code symbols transmitted in the generic frame are

$$c_i(\ell) = \begin{cases} b_i(1), & \ell = 1 \\ b_i(2) + s^{(i)}_{\mu(i-1,3)}(1), & \ell = 2 \\ s^{(i)}_{\mu(i-1,3)}(2) + s^{(i)}_{\mu(i-2,3)}(1), & \ell = 3 \end{cases} \tag{2}$$

from which the simple structure of the single user is evident. Diversity is obtained because symbols travel through different channels. Also there is no codeword at the user location; codewords are confined at the BS after collecting users' transmissions. Avoiding channel coding and decoding at user location leads to flexibility and simplicity in rate and diversity, without dealing with sub-codes or puncturing [8], still keeping the advantage of coding with respect to repetition. Table I shows the transmitted code symbols during the generic frame, when error-free symbol detection at user location is assumed ($s^{(i)}_j(\ell) = c_j(\ell)$). It can be noted how each source symbol is transmitted over two different paths, thus intuitively achieving diversity of order 2. A more precise computation of the achieved diversity order will be given in Section III.

The overall code at the BS is described as

$$\begin{bmatrix} c_1(1) \\ c_2(1) \\ c_3(1) \\ c_1(2) \\ c_2(2) \\ c_3(2) \\ c_1(3) \\ c_2(3) \\ c_3(3) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} b_1(1) \\ b_2(1) \\ b_3(1) \\ b_1(2) \\ b_2(2) \\ b_3(2) \end{bmatrix}.$$

[2]Though the analysis is not restricted to this case, we only consider $n = J$ to focus on diversity gain. As it will be clear, $n \ge J$ is required: $n > J$ increases space dimension in (1), thus better separation among codewords, but the same number of channels makes some "dimensions" are simultaneously scaled with the same fading coefficient; diversity is NOT increased.

TABLE I
CODE SYMBOLS FOR THE CODE (2, 3) WITH ERROR-FREE DETECTION AT USER LOCATION

user \ ST	1	2	3
1	$b_1(1)$	$b_1(2) + b_3(1)$	$b_3(2)$
2	$b_2(1)$	$b_2(2) + b_1(1)$	$b_1(2)$
3	$b_3(1)$	$b_3(2) + b_2(1)$	$b_2(2)$

TABLE II
CODES FOR ($R = 2/3$, $L = 2$) AND ($R = 2/5$, $L = 3$) WITH $M = 2$

$\mathbf{u}^{\mathrm{T}}$	$\mathbf{v}^{\mathrm{T}}$	$\mathbf{u}^{\mathrm{T}}$	$\mathbf{v}^{\mathrm{T}}$
00	000	00	00000
01	011	01	01101
10	110	10	11011
11	101	11	10110

The rows of the encoding matrix presenting 1's in the same positions may be grouped separately, as every symbol is related always to the same set of source symbols. More specifically, rows (1, 5, 9), (2, 6, 7), and (3, 4, 8) may be grouped; then (1) is easily recognized, denoting: $\mathbf{u}_i^{\mathrm{T}} = [b_i(1), b_{\mu(i+1,3)}(2)]$, $\mathbf{v}_i^{\mathrm{T}} = [c_i(1), c_{\mu(i+1,3)}(2), c_{\mu(i+2,3)}(3)]$, $\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$. Decoupling the overall code equations into (1) allows the BS to decode by computation of 12 distances in a three-dimensional space instead of 64 distances in a nine-dimensional space. *Maximum likelihood* (ML) decoding at the BS is performed via knowledge of $\mathbf{A}$ as (1) gives Table II. The minimum distance of the code in Table II is $d_{\min} = 2$.

B. Extending the Basic Example

The proposed algorithm can be extended to arbitrary $J = n$ and $k = n - 1$ with $M = 2$. The code is described by a simple structure at user location, corresponding to a parity-check symbol transmission in the nth ST: $c_i(\ell) = \begin{cases} b_i(1) & \ell = 1 \\ b_i(\ell) + s^{(i)}_{\mu(i-1,n)}(\ell - 1) & \ell = 2, \ldots, n-1 \\ \sum_{j=1}^{n-1} s^{(i)}_{\mu(i-n+j,n)}(j) & \ell = n \end{cases}$, and (1) is recognized via: $\mathbf{u}_i^{\mathrm{T}} = [b_i(1), b_{\mu(i+1,n)}(2), \ldots, b_{\mu(i+n-2,n)}(n-1)]$, $\mathbf{v}_i^{\mathrm{T}} = [c_i(1), c_{\mu(i+1,n)}(2), \ldots, c_{\mu(i+n-1,n)}(n)]$, $\mathbf{A}^{\mathrm{T}} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 & \mathrm{mod}(n-1,2) \\ 0 & 1 & 1 & \ldots & 1 & \mathrm{mod}(n-2,2) \\ 0 & 0 & 1 & \ldots & 1 & \mathrm{mod}(n-3,2) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & 1 \end{bmatrix}$. The computation saving at the BS for this case is replacing $2^{n(n-1)}$ distances in an n^2-dimensional space with $n2^{n-1}$ distances in an n-dimensional space. Again we have $d_{\min} = 2$.

A scheme with the above-described characteristics (in terms of computational complexity savings) can be implemented for a generic rate $R = k/n$ via a linear block code (k, n). Unfortunately, it is not easy to find a general formula including all the cases with arbitrary values for k and n. We show how simple algorithms based on circular ordering of the users allow to exploit linear block codes.

Notice that $k = n - 2$ is not very appropriate as codes $(n-2, n)$ still have $d_{\min} = 2$. Considering $k = n-3$, we have codes

$(n-3,n)$ with $d_{\min}=3$ for $n=5,6,7$, that may be used along the same lines of Section II-A.

The code (2, 5) with $R=2/5$ and $d_{\min}=3$ is given by:

$$c_i(\ell) = \begin{cases} b_i(1), & \ell=1 \\ b_i(2)+s^{(i)}_{\mu(i-1,5)}(1), & \ell=2 \\ s^{(i)}_{\mu(i-1,5)}(\ell-1)+s^{(i)}_{\mu(i-2,5)}(\ell-2), & \ell=3,4,5, \end{cases}$$

and (1) is recognized via: $\mathbf{u}_i^{\mathrm{T}}=[b_i(1),b_{\mu(i+1,5)}(2)]$,

$\mathbf{v}_i^{\mathrm{T}}=[c_i(1),c_{\mu(i+1,5)}(2),c_{\mu(i+2,5)}(3),c_{\mu(i+3,5)}(4),c_{\mu(i+4,5)}(5)]$

$\mathbf{A}^{\mathrm{T}}=\begin{bmatrix}1&1&0&1&1\\0&1&1&0&1\end{bmatrix}$. The code is shown in Table II.

The code (3, 6) with $R=1/2$ and $d_{\min}=3$ is given by: $c_i(\ell)=\begin{cases} b_i(1), & \ell=1 \\ b_i(\ell)+s^{(i)}_{\mu(i-1,6)}(\ell-1), & \ell=2,3 \\ s^{(i)}_{\mu(i-2,6)}(\ell-2)+s^{(i)}_{\mu(i-3,6)}(\ell-3), & \ell=4,5,6 \end{cases}$ and (1) is recognized via: $\mathbf{u}_i^{\mathrm{T}}=[b_i(1),b_{\mu(i+1,6)}(2),b_{\mu(i+2,6)}(3)]$,

$$\mathbf{v}_i^{\mathrm{T}}=[c_i(1),c_{\mu(i+1,6)}(2),c_{\mu(i+2,6)}(3),c_{\mu(i+3,6)}(4), c_{\mu(i+4,6)}(5),c_{\mu(i+5,6)}(6)]$$

$$\mathbf{A}^{\mathrm{T}}=\begin{bmatrix}1&1&1&0&0&1\\0&1&1&1&0&0\\0&0&1&0&1&1\end{bmatrix}.$$

The code (4, 7) with $R=4/7$ and $d_{\min}=3$ is

$$c_i(\ell)=\begin{cases} b_i(1), & \ell=1 \\ b_i(\ell)+s^{(i)}_{\mu(i-1,7)}(\ell-1), & \ell=2,3,4 \\ s^{(i)}_{\mu(i-2,7)}(\ell-2)+s^{(i)}_{\mu(i-3,7)}(\ell-3) & \\ +s_{\mu(i-4,7),i}(\ell-4), & \ell=5,6,7 \end{cases}$$

and (1) is recognized via: $\mathbf{u}_i^{\mathrm{T}}=[b_i(1),b_{\mu(i+1,7)}(2),b_{\mu(i+2,7)}(3),b_{\mu(i+3,7)}(4)]$,

$$\mathbf{v}_i^{\mathrm{T}}=[c_i(1),c_{\mu(i+1,7)}(2),c_{\mu(i+2,7)}(3),c_{\mu(i+3,7)}(4), c_{\mu(i+4,7)}(5),c_{\mu(i+5,7)}(6),c_{\mu(i+6,7)}(7)],$$

$$\mathbf{A}^{\mathrm{T}}=\begin{bmatrix}1&1&1&1&1&1&1\\0&1&1&1&0&1&0\\0&0&1&1&1&0&1\\0&0&0&1&0&1&1\end{bmatrix}.$$

In general, the code may be obtained via a matrix having in column: 1) a lower triangular matrix; and 2) a parity-check matrix from recursive combination of previous rows. The computation saving at the BS is replacing 2^{nk} distances in an n^2-dimensional space with $n2^k$ distances in an n-dimensional space.

C. Arbitrary Alphabet Size

Results are still valid for a general M-ary alphabet when considering modulo-M sum. In the case $M=4$, $R=2/3$, use of (2) gives the code shown in Table III, in which (1) is recognized via: $\mathbf{u}_i^{\mathrm{T}}=[b_i(1),b_{\mu(i+1,3)}(2)]$, $\mathbf{v}_i^{\mathrm{T}}=[c_i(1),c_{\mu(i+1,3)}(2),c_{\mu(i+2,3)}(3)]$, $\mathbf{A}^{\mathrm{T}}=\begin{bmatrix}1&1&2\\0&1&1\end{bmatrix}$.

TABLE III
CODE FOR ($R=2/3$, $L=2$) WITH $M=4$

$\mathbf{u}^{\mathrm{T}}$	$\mathbf{v}^{\mathrm{T}}$	$\mathbf{u}^{\mathrm{T}}$	$\mathbf{v}^{\mathrm{T}}$	$\mathbf{u}^{\mathrm{T}}$	$\mathbf{v}^{\mathrm{T}}$	$\mathbf{u}^{\mathrm{T}}$	$\mathbf{v}^{\mathrm{T}}$
00	000	10	110	20	220	30	330
01	011	11	121	21	231	31	301
02	022	12	132	22	202	32	312
03	033	13	103	23	213	33	323

III. PERFORMANCE

We derive analytical bounds for the BER of users in a CG implementing the proposed scheme and relate the achieved order of diversity, defined as the negative slope of the log BER versus log SNR curve, to the minimum distance of the considered linear block code. We assume M-PSK with coherent detection, with one-to-one mapping of code symbols into modulation symbols (with abuse of notation, we use $c_i(l)$ for both). Users and the BS are assumed perfectly synchronized, and all channels are *additive white Gaussian noise* with "quasi-static" fading.

The baseband discrete-time signal (after matched filtering and sampling at the symbol rate) that the jth user receives ($j=0$ for BS) from the ith user on the ℓth ST of a frame is

$$y_{i,j}(\ell)=\alpha_{i,j}\sqrt{R\mathcal{E}_b}\,c_i(\ell)+w_{i,j}(\ell), \qquad \begin{matrix}1\le\ell\le n\\0\le j\le J\\1\le i\le J\end{matrix} \tag{3}$$

where $\mathcal{E}_b$ is the energy-per-source-symbol of the single user, $\alpha_{i,j}\sim Rayleigh(\varsigma^2_{i,j})$ and $w_{i,j}(l)\sim\mathcal{N}(0,\sigma^2_{i,j})$ are the fading envelope and the additive noise over the channel between users i and j. We consider a symmetric scenario, i.e., independent identically distributed (*iid*) channels among users and between users and BS, $\varsigma^2_{i,j}=\begin{cases}\varsigma^2_o & j=0\\ \varsigma^2_u & j\neq 0,i\end{cases}$, $\sigma^2_{i,j}=\begin{cases}\eta_o/2 & j=0\\ \eta_u/2 & j\neq 0,i\end{cases}$. Equations (1) and (3) give

$$\mathbf{r}_i=\sqrt{R\mathcal{E}_b}\,\mathrm{Diag}(\mathbf{f}_i)\cdot(\mathbf{v}_i)+\mathbf{z}_i, \quad 1\le i\le J \tag{4}$$

where $\mathbf{r}_i$, $\mathbf{f}_i$, and $\mathbf{z}_i$ represent the signals received at the BS, and the corresponding fading and noise effects. Equation (4) represents the proposed code over the considered channel. The effect of fading is scaling independently each dimension.

A. BER With Error-Free Symbol Detection at User Location

We first focus on the scheme analyzed in Section II-A, neglecting the presence of fading. In absence of fading, by use of the union bound [1], the BER can be approximated as $(3/2)\mathcal{Q}\left(2\sqrt{R\mathcal{E}_b/\eta_o}\right)$, where $\mathcal{Q}(t)=1/\pi\int_0^{\pi/2}\exp\left(-(t^2/2\sin^2\theta)\right)d\theta$. In presence of fading, the set of distances in the signal-space is modified by the fading coefficients, and the former expression provides the conditional BER to be averaged according to fading statistics

$$P_{e/\gamma}\approx\frac{3}{2}\mathcal{Q}(\sqrt{2\gamma}) \tag{5}$$

where $\gamma=(\alpha^2_{i0}+\alpha^2_{j0})(R\mathcal{E}_b/\eta_o)$ with $i\neq j$. For *iid* Rayleigh fading coefficients, the probability density function of γ is

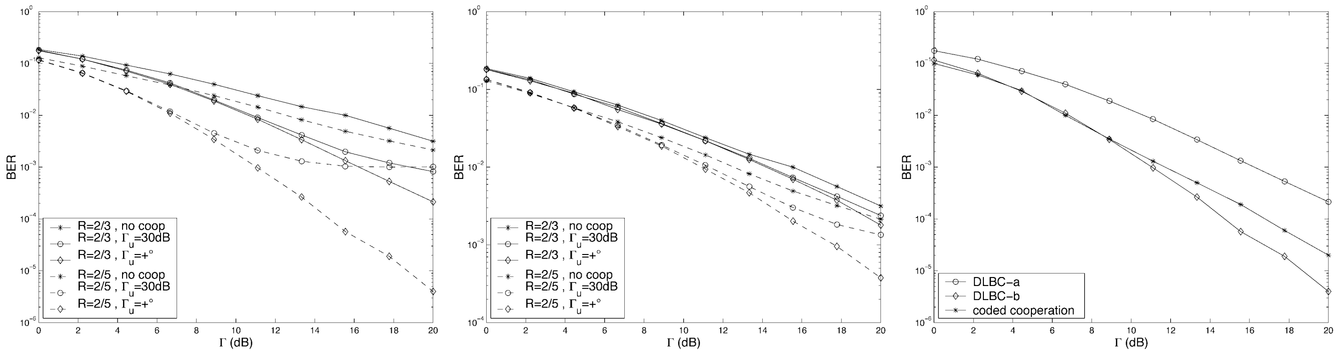
$$p(\gamma)=\frac{\gamma}{\Gamma^2}\exp\left(-\frac{\gamma}{\Gamma}\right)\ (\gamma>0) \tag{6}$$

where $\Gamma=(2\varsigma^2_o R\mathcal{E}_b/\eta_o)$ is the average SNR at the BS, defined as the ratio between the average received energy-per-source-symbol and the one-sided noise power spectral density. BER, from (5) and (6), is

$$\frac{3}{2}\int_0^{+\infty}\frac{\gamma}{\Gamma^2}\exp\left(-\frac{\gamma}{\Gamma}\right)\mathcal{Q}(\sqrt{2\gamma})d\gamma=\frac{3}{2\pi\Gamma^2}\int_0^{\pi/2}\frac{1}{\left(\frac{1}{\Gamma}+\frac{1}{\sin^2\theta}\right)^2}d\theta$$

upper-bounded with $P_e\approx 3/4(1+\Gamma)^{-2}$.

For the general case, $R=k/n$, $\gamma\sim Erlang(\Gamma,d_{\min})$ [1], where $d_{\min}$ is the minimum distance of the considered linear block code. Similar derivation, replacing (6) with $p(\gamma)=\gamma^{d_{\min}-1}e^{-\gamma/\Gamma}/(d_{\min}-1)!\Gamma^{d_{\min}}$ $(\gamma>0)$, gives $P_e(\Gamma)\approx(2^k-1)/4(1+\Gamma)^{-d_{\min}}$, i.e., order of diversity $L=d_{\min}\le n-k+1$ is obtained.



(a) (b) (c)

Fig. 1. Performance for BPSK with coherent detection. (a) Independent fading. (b) Correlated fading. (c) Comparison with coded cooperation.

B. Effects of Erroneous Symbol Detection at User Location

Consider a more realistic scenario where Γ_u and Γ represent the average SNR between two users and between each user and the BS. We assume Γ_u and Γ independent: Γ_u is constant and Γ is increasing. It may represent a vehicle (BS) approaching a static sensor (user) network. Each user has to detect side symbols in order to build the code symbols for transmission to the BS. Users detect erroneously side symbols with probability [1] $P_\varepsilon = (1/2(1+\Gamma_u))$ due to the assumption for user–user channels. The presence of errors in the user–user channels can destroy the equivalence between (1) and (4). In such a case, the signal constellation used for ML decoding is not valid, and all the vertices of the n-dimensional hypercube with side length $2\sqrt{R\mathcal{E}_b}$ should be taken into account.

In the basic example, the constellation is valid with probability $P_{\rm vc} = (1-P_\varepsilon)^2\left((1-P_\varepsilon)^2 + 3P_\varepsilon^2\right)$. Denoting P_e and P_E the BER for the error-free and error-presence cases, then

$$\begin{aligned}1 - P_E = \Pr(\mathrm{c}) &= P_{\rm vc}\Pr(\mathrm{c}|\mathrm{vc}) + (1-P_{\rm vc})\Pr(\mathrm{c}|\text{not vc})\\ &= P_{\rm vc}(1-P_e) + (1-P_{\rm vc})\Pr(\mathrm{c}|\text{not vc}).\end{aligned}$$

Assuming $\Pr(\mathrm{c}|\text{not vc}) \approx (1/3)P_e$, we get

$$P_E \approx (1-P_{\rm vc}) - \frac{1}{3}(1-4P_{\rm vc})P_e \approx \frac{3}{\Gamma_u+1} + \frac{3}{4}\frac{\Gamma_u - 1}{\Gamma_u+3}(1+\Gamma)^{-2}.$$

The approximation shows the presence of an error floor depending on the average SNR on the user–user channel.[3] In general, the scheme obtains the claimed order of diversity only for a limited range of average SNR, bounded by the (worst) average user–user SNR. Performance in terms of BER versus average SNR can be roughly expressed as $P_E(\Gamma) = P_e(\Gamma)(\Gamma < \Gamma_u) + P_e(\Gamma_u)(\Gamma > \Gamma_u)$. We obtain spatial diversity in a range upper-bounded by the average SNR among cooperative users.

IV. Simulations

Simulations have been performed with Matlab. BPSK modulation[4] with uniform *a priori* symbol distribution, and coherent detection on each channel are considered. ML decoding at the BS is performed by selection of the minimum distance in the signal-space constellation. Channels are statistically *iid*, and fading coefficients are constant within the frame.

Results confirm the claimed diversity in the error-free case. The performance follows the error-free trend when the average SNR does not exceed the average user–user SNR and shows the expected error floor. Fig. 1(a) shows the performance for $R = 2/3$, 2/5 with independent fading coefficients among frames. The results corresponds to the cases where users do NOT cooperate, and cooperate with inter–user SNR equal to $\Gamma_u = 30$ dB, $+\infty$. No-cooperation needs $\mathrm{SNR} = 23$ dB to achieve $\mathrm{BER} = 10^{-3}$, while cooperation among three and five users (with error-free detection at user location) provides a reduction of 7 and 12 dB, respectively. Fig. 1(b) considers correlated fading coefficients among frames according to Jakes model [2] with a normalized Doppler frequency of 0.1. The energy saving reduces to 2 and 5 dB.

Fig. 1(c) compares (in the error-free case) the coded-cooperation [8] with $R = 1/4$, $L = 2$, and the two proposed schemes: DLBC-a with $R = 2/3$, $L = 2$, DLBC-b with $R = 2/5$, $L = 3$. Both the proposed schemes achieve the same BER at higher rates, wasting fewer resources to create cooperative diversity. To give an idea of the operating conditions, we note that to achieve a $\mathrm{BER} = 10^{-3}$: 1) coded cooperation uses 2 users with $R = 1/4$ at 12 dB; 2) DLBC-a uses 3 users with $R = 2/3$ at 16 dB; 3) DLBC-b uses 5 users with $R = 2/5$ at 11 dB.

V. Conclusion

A novel scheme for coded cooperative wireless communications has been proposed, with flexible trade-off between spatial diversity and transmission rates by distributing information of all users over all channels and requiring one transmission channel per user. An appropriate cooperative algorithm keeps computational complexity at the BS low. Analysis and simulations show an error floor on the performance; thus, the choice cooperative users is critical, aiming at good user-user channels.

[3]Cooperation among users with channels conditions worse than the channel to the BS results in unacceptable waste of resource.

[4]Analogous results have been obtained with QPSK modulation.

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